

Numericals:

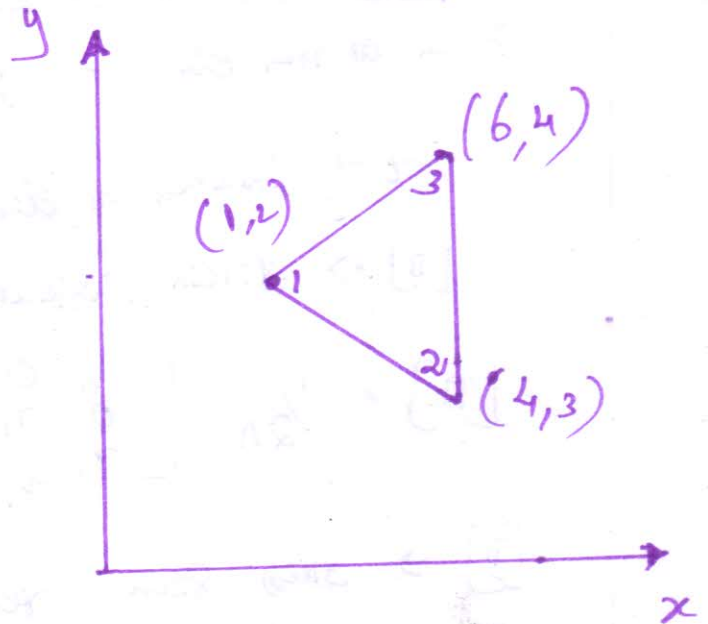
- 1) The Node Co-ordinates of the triangular element are shown in figure. At the interior point P, the x-co-ordinate is 3.5 and $N_1 = 0.4$, Calculate N_2, N_3 and the y-co-ordinates at point P.

Soln:

$$x_1, x_2, x_3 = 1, 4, 6$$

$$y_1, y_2, y_3 = 2, 3, 6$$

$$x = 3.5, N_1 = 0.4$$



We know that

$$N_1 + N_2 + N_3 = 1$$

$$0.4 + N_2 + N_3 = 1 \Rightarrow (N_2 + N_3 = 0.6) \rightarrow \textcircled{1}$$

$$x = N_1 x_1 + N_2 x_2 + N_3 x_3$$

$$3.5 = 0.4(1) + N_2(4) + N_3(6)$$

$$\boxed{3.1 = 4N_2 + 6N_3} \rightarrow \textcircled{2}$$

Resolving $\textcircled{1}$ & $\textcircled{2}$ we get.

$$N_3 = 0.35, N_2 = 0.25$$

$$y = N_1 y_1 + N_2 y_2 + N_3 y_3$$

$$y = 0.4 \times 2 + 0.25 \times 3 + 0.35 \times 4$$

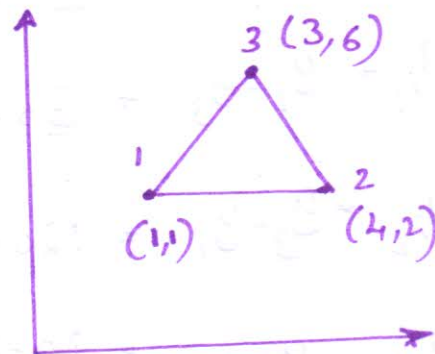
$$y = 2.95$$

HW
②

Determine the x, y Co-ord at point P for the A^h element shown. The shape for N_1 & N_2 are 0.2 & 0.3 respectively.

$$N_3 = 0.5 \quad (N_1 + N_2 + N_3 = 1)$$

$$x = 2.9, y = 3.8$$



③ Determine the Stiffness matrix for the CST element shown in fig. The Co-ordinates are given in units of millimeter. Assume

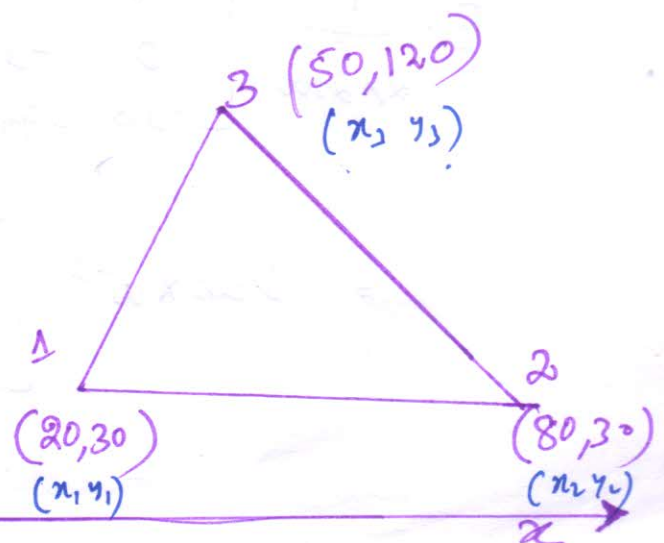
Plane Stress Condition.

tan

$$E = 210 \text{ GPa.}$$

$$\mu = 0.25$$

$$t = 6 \text{ mm.}$$



$$[K] = [B]^T [D] [B] \cdot A \cdot t$$

$$A = \frac{1}{2} \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 1 & 20 & 30 \\ 1 & 80 & 30 \\ 1 & 50 & 120 \end{vmatrix}$$

$$A = 2700 \text{ mm}^2$$

$$[B] = \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

$$q_1 = y_2 - y_3 = 30 - 120 = -90$$

$$q_2 = y_3 - y_1 = 120 - 30 = 90$$

$$q_3 = y_1 - y_2 = 30 - 30 = 0$$

$$r_1 = x_3 - x_2 = 50 - 80 = -30$$

$$r_2 = x_1 - x_3 = 20 - 50 = -30$$

$$r_3 = x_2 - x_1 = 80 - 20 = 60$$

$$B = \frac{1}{2 \times 2700} \begin{bmatrix} -90 & 0 & 90 & 0 & 0 & 0 \\ 0 & -30 & 0 & -30 & 0 & 60 \\ -30 & -90 & -30 & 90 & 60 & 0 \end{bmatrix}$$

$$= 5.555 \times 10^{-3} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

Stress Strain Relation, [D]. Plan stress problem. (4)

$$[D] = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}$$

$$= \frac{2.1 \times 10^5}{1 - (0.25)^2} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{bmatrix}$$

$$[D] = 56 \times 10^3 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[D][B] = 56 \cdot 10^3 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \times 5.555 \times 10^{-3} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$= 311.08 \begin{bmatrix} -12 & -1 & 12 & -1 & 0 & 2 \\ -3 & -4 & 3 & -4 & 0 & 8 \\ -1.5 & -4.5 & -1.5 & 4.5 & 3 & 0 \end{bmatrix}$$

$$[B]^T = 5.555 \times 10^{-3} \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

$$[B]^T [D] [B] = 5.55 \times 10^{-3} \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \times 311.08 \begin{bmatrix} -12 & \dots & 2 \\ -3 & \dots & 8 \\ -1.5 & \dots & 3.0 \end{bmatrix}$$

$$\underline{[B]^T [D] [B]} = 1.728 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

$$[K] = \underline{[B]^T [D] [B]} A L$$

$$= 1.728 \left[\begin{array}{c} \text{Matrix} \end{array} \right] \times 2700 \times 10^3$$

Stiffness matrix

$[K]$

$$= 46.656 \times 10^3$$

$$\begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

4

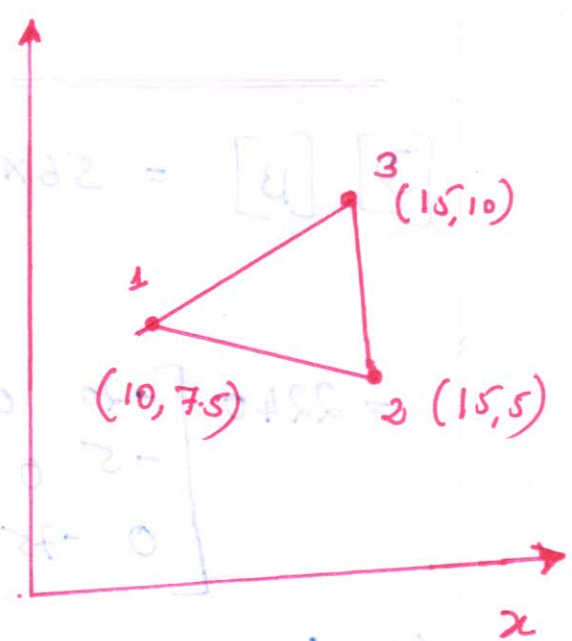
Calculate the element stresses $\sigma_x, \sigma_y, \tau_{xy}, \sigma_1$ and σ_2 and the principle angle θ_p for the element shown in fig (i)

The nodal displacements are

$$\begin{aligned} u_1 &= 2.0 \text{ mm} & v_1 &= 1.0 \text{ mm} \\ u_2 &= 0.5 \text{ mm} & v_2 &= 0.0 \text{ mm} \\ u_3 &= 3.0 \text{ mm} & v_3 &= 1.0 \text{ mm} \end{aligned}$$

Take $E = 2.1 \times 10^5 \text{ N/mm}^2$ and $\mu = 0.25$

Assume plane stress condition.



Soln:

Area of the element $A = \frac{1}{2} \begin{vmatrix} 1 & 10 & 7.5 \\ 1 & 15 & 5 \\ 1 & 15 & 10 \end{vmatrix}$

$A = 12.5 \text{ mm}^2$

$$[B] = \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & q_1 & \gamma_2 & q_2 & \gamma_3 & q_3 \end{bmatrix}$$

$$= \frac{1}{12.5 \times 2} \begin{bmatrix} -5 & 0 & 2.5 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & -5 & 0 & 5 \\ 0 & -5 & -5 & 2.5 & 5 & 2.5 \end{bmatrix}$$

$$= 0.04 \begin{bmatrix} -5 & 0 & 2.5 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & -5 & 0 & 5 \\ 0 & -5 & -5 & 2.5 & 5 & 2.5 \end{bmatrix}$$

$$[D] = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix} = \frac{2.1 \times 10^5}{1-0.25^2} \begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{bmatrix}$$

$$[D] = 56 \times 10^3 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[D][B] = 56 \times 10^3 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \times 0.04 \begin{bmatrix} -5 & 0 & 25 & 0 & 25 & 0 \\ 0 & 0 & 0 & -5 & 0 & 5 \\ 0 & -5 & -5 & 25 & 5 & 25 \end{bmatrix}$$

$$= 2240 \begin{bmatrix} -20 & 0 & 10 & -5 & 10 & 5 \\ -5 & 0 & 2.5 & -20 & 2.5 & 20 \\ 0 & -7.5 & -7.5 & 3.75 & 7.5 & 3.75 \end{bmatrix}$$

We know that .

$$\{\sigma\} = [D][B]\{u\} = [D][B] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

$$= 2240 \begin{bmatrix} -20 & 0 & 10 & -5 & 10 & 5 \\ -5 & 0 & 2.5 & -20 & 2.5 & 20 \\ 0 & -7.5 & -7.5 & 3.75 & 7.5 & 3.75 \end{bmatrix} \begin{Bmatrix} 2.0 \\ 1.0 \\ 0.5 \\ 0 \\ 3.1 \\ 1.0 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = 2240 \begin{Bmatrix} 0 \\ 18.75 \\ 7.5 \end{Bmatrix} = 10^3 \begin{Bmatrix} 0 \\ 42 \\ 33.6 \end{Bmatrix}$$

Max Normal Stress $\sigma_{max} = \sigma_1$

$$= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + 42 \times 10^3}{2} + \sqrt{\left(\frac{0 - 42 \times 10^3}{2}\right)^2 + (33.6 \times 10^3)^2}$$

$$= 21000 + 39622.72 \Rightarrow \underline{\underline{\sigma_1 = 60.62 \times 10^3 \text{ N/mm}^2}}$$

Minimum normal stress $\sigma_{\min} = \sigma_2$

$$= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{0 - 42 \times 10^3}{2}\right)^2 + (33.6 \times 10^3)^2}$$

$$= 21000 - 39622.72 \Rightarrow \underline{\underline{\sigma_2 = -18.62 \times 10^3 \text{ N/mm}^2}}$$

Principal angle, $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$$2\theta_p = \tan^{-1} \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right)$$

$$= \tan^{-1} \left(\frac{2 \times 33.6 \times 10^3}{0 - 42 \times 10^3} \right) = \tan^{-1} (-1.6)$$

$$\theta_p = -28.99^\circ$$

$$\underline{\underline{\theta_p \approx 30^\circ}}$$

⑤ The 2D propped beam shown in fig 5. divided into two CST elements. Determine the nodal displacements and element stresses using plane stress conditions. Body force is neglected in comparison with the external forces.

Take, thickness $t = 10\text{mm}$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

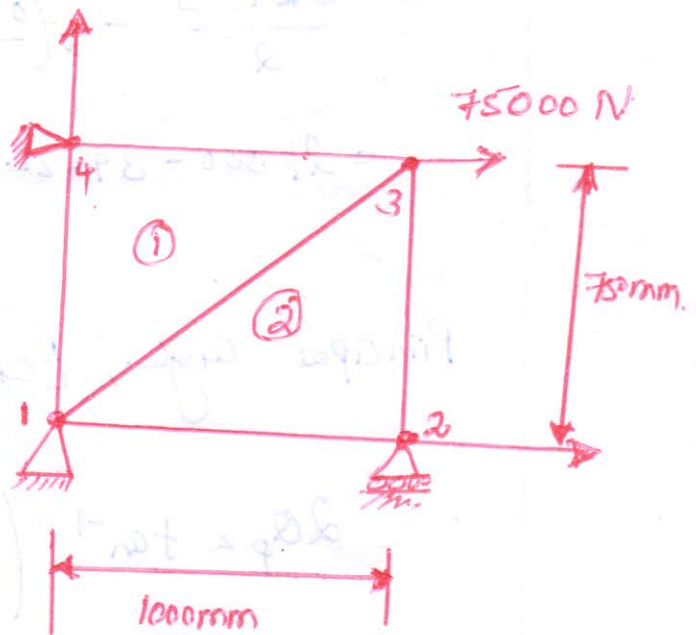
$$\mu = 0.25$$

Soln:

$$\text{Node 1 } (x_1, y_1) = (0, 0)$$

$$\text{Node 3 } (x_2, y_2) = (1000, 750)$$

$$\text{Node 4 } (x_3, y_3) = (0, 750)$$



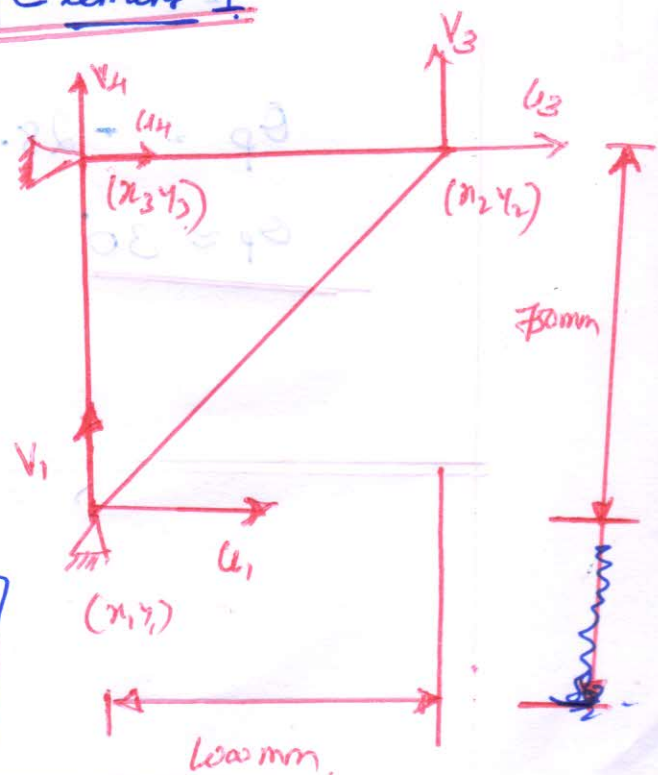
Element 1

$$[K] = [B]^T [D] [B] A \cdot t$$

$$A = \frac{1}{2} \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1000 & 750 \\ 1 & 0 & 750 \end{vmatrix}$$

$$= 375 \times 10^3 \text{ mm}^2$$

$$[B] = \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$



$$[B] = \frac{1}{2 \times 375 \times 10^3} \begin{bmatrix} 0 & 0 & 750 & 0 & -750 & 0 \\ 0 & -1000 & 0 & 0 & 0 & 1000 \\ -1000 & 0 & 0 & 750 & 1000 & -750 \end{bmatrix}$$

$$= \frac{250}{750 \times 10^3} \begin{bmatrix} 0 & 0 & 3 & 0 & -3 & 0 \\ 0 & -4 & 0 & 0 & 0 & 4 \\ -4 & 0 & 0 & 3 & 4 & -3 \end{bmatrix}$$

$$[D] = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix} = 2 \times 10^5 \times 0.2667 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[D][B] = \frac{250}{750 \times 10^3} \times 2 \times 10^5 \times 0.2667 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix} \begin{bmatrix} 0 & 0 & 30 & -30 \\ 0 & -4 & 0 & 0 & 4 \\ -4 & 0 & 0 & 3 & 4 & -3 \end{bmatrix}$$

$$= 17.78 \begin{bmatrix} 0 & -4 & 12 & 0 & -12 & 4 \\ 0 & -16 & 3 & 0 & -3 & 16 \\ -6 & 0 & 0 & 4.5 & 6 & -4.5 \end{bmatrix}$$

$$[B]^T = \frac{250}{750 \times 10^3} \begin{bmatrix} 0 & 0 & -4 \\ 0 & -4 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 3 \\ -3 & 0 & 4 \\ 0 & 4 & -3 \end{bmatrix}$$

$$[B]^T [D] [B] =$$

$$= 5.927 \times 10^{-3} \begin{bmatrix} 24 & 0 & 0 & -18 & -24 & 18 \\ 0 & 64 & -12 & 0 & 12 & -64 \\ 0 & -12 & 36 & 0 & -36 & 12 \\ -18 & 0 & 0 & 13.5 & 18 & -13.5 \\ -24 & 12 & -36 & 18 & 60 & -30 \\ 18 & -64 & 12 & -13.5 & -30 & 77.5 \end{bmatrix}$$

$$[K]_1 = [B]^T [D] [B] \times A \times t$$

$$= 2.22 \times 10^4 \begin{bmatrix} 24 & \dots & 18 \\ \vdots & & \\ 18 & & 77.5 \end{bmatrix}$$

$$= 1 \times 10^4 \begin{bmatrix} \begin{matrix} u_1 & v_1 & u_2 & v_2 & u_4 & v_4 \end{matrix} \\ \begin{matrix} 53.28 & 0 & 0 & -39.96 & -53.28 & 39.96 \end{matrix} \\ \begin{matrix} 0 & 142.08 & -26.64 & 0 & 26.64 & -142.08 \end{matrix} \\ \begin{matrix} 0 & -26.64 & 79.92 & 0 & -79.92 & 26.64 \end{matrix} \\ \begin{matrix} -39.96 & 0 & 0 & 29.97 & 39.96 & -29.97 \end{matrix} \\ \begin{matrix} -53.28 & 26.64 & -79.92 & 39.96 & 133.2 & -66.6 \end{matrix} \\ \begin{matrix} 39.96 & -142.08 & 26.64 & -29.97 & -166.6 & 172.05 \end{matrix} \end{bmatrix} \begin{matrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_4 \\ v_4 \end{matrix}$$

ELEMENT: 2

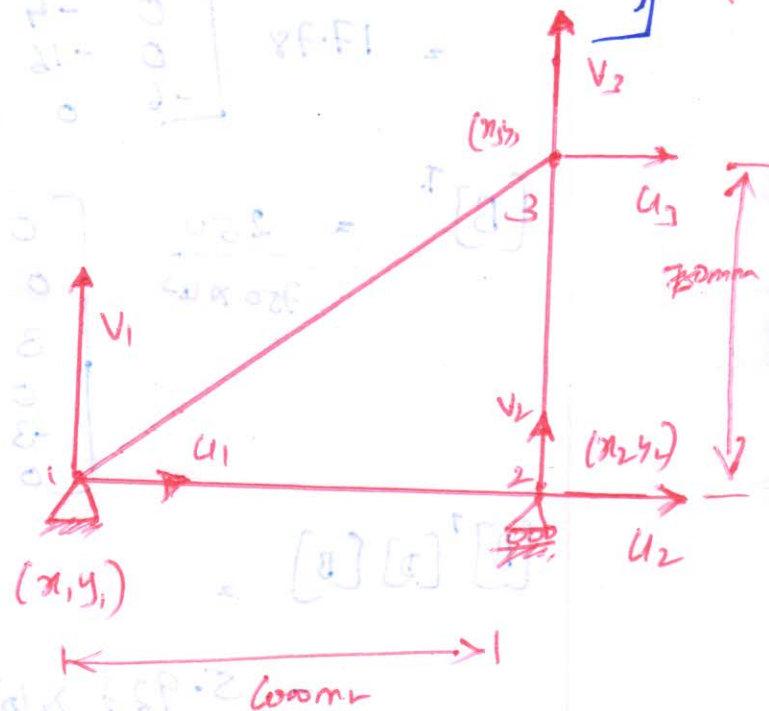
Node 1 (0,0)

Node 2 (1000,0)

Node 3 (1000,750)

$$A = \frac{1}{2} \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1000 & 0 \\ 1 & 1000 & 750 \end{vmatrix}$$

$$= 375 \times 10^3 \text{ mm}^2$$



$$[B] = \frac{250}{2 \times 375 \times 10^3} \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 4 \\ 0 & -3 & -4 & 3 & 4 & 0 \end{bmatrix}$$

$$[D] = \frac{E}{(1-\mu^2)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix} = \frac{2 \times 10^5 \times 0.25}{0.9375} \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[D][B] = 17.78 \begin{bmatrix} -12 & 0 & 12 & -4 & 0 & 4 \\ -3 & 0 & 3 & -16 & 0 & 16 \\ 0 & -4.5 & -6 & 4.5 & 6 & 0 \end{bmatrix}$$

$$B^T = \frac{250}{750 \times 10^3} \begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & -3 \\ 3 & 0 & -4 \\ 0 & -4 & 3 \\ 0 & 0 & 4 \\ 0 & 4 & 0 \end{bmatrix}$$

$$[B]^T [D] [B] = \frac{250}{750 \times 10^3} \times 17.78 \begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & -3 \\ 3 & 0 & -4 \\ 0 & -4 & 3 \\ 0 & 0 & 4 \\ 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} -12 & 0 & 12 & -4 & 0 & 4 \\ -3 & 0 & 3 & -16 & 0 & 16 \\ 0 & -4.5 & -6 & 4.5 & 6 & 0 \end{bmatrix}$$

$$= 5.927 \times 10^{-3} \begin{bmatrix} 36 & 0 & -36 & 12 & 0 & -12 \\ 0 & 13.5 & 18 & -13.5 & -18 & 0 \\ -36 & 18 & 60 & -30 & -24 & 12 \\ 12 & -13.5 & -30 & 37.5 & 18 & -64 \\ 0 & -18 & -24 & 18 & 24 & 0 \\ 12 & 0 & 12 & -64 & 0 & 64 \end{bmatrix}$$

$$[K]_2 = [B]^T [D] [B] \times 10^4$$

$$= 5.927 \times 10^4 \begin{bmatrix} 79.92 & 0 & -79.91 & 26.64 & 0 & -26.64 \\ 0 & 29.97 & 39.96 & -29.97 & -39.96 & 0 \\ -79.92 & 39.96 & 133.2 & -66.6 & -53.28 & 26.64 \\ 26.64 & -29.97 & -66.6 & 172.05 & 39.96 & -142.1 \\ 0 & -39.96 & -53.28 & 39.96 & 53.28 & 0 \\ -26.64 & 0 & 26.64 & -142.08 & 0 & 142.1 \end{bmatrix} \begin{matrix} u_1 \\ u_2 \\ u_3 \\ v_1 \\ v_2 \\ v_3 \end{matrix}$$

Global Stiffness matrix $[K]$

$$\begin{array}{c}
 \begin{array}{cccccccc}
 & u_1 & v_1 & u_2 & v_2 & u_3 = v_3 & u_4 & v_4 \\
 \begin{bmatrix}
 133.2 & 0 & -79.92 & 26.64 & 0 & -66.6 & -53.28 & 39.96 \\
 0 & 172.05 & 36.96 & -29.97 & -66.6 & 0 & 26.64 & -142.01 \\
 -79.92 & 39.96 & 133.2 & -66.6 & -53.28 & 26.64 & 0 & 0 \\
 26.64 & -29.97 & -66.6 & 172.05 & ~~-53.28~~ & ~~39.96~~ & -142.01 & 0 \\
 0 & -66.6 & -53.28 & 39.96 & 133.2 & 0 & -79.92 & 26.64 \\
 -66.6 & 0 & 26.64 & -142.08 & 0 & 172.05 & 39.96 & -29.97 \\
 -53.28 & 26.64 & 0 & 0 & -79.92 & 39.96 & 133.2 & -66.6 \\
 39.96 & -142.08 & 0 & 0 & 26.64 & -29.97 & -66.6 & 172.05
 \end{bmatrix}
 \end{array}
 \end{array}
 \begin{array}{l}
 u_1 \\
 v_1 \\
 u_2 \\
 v_2 \\
 u_3 \\
 v_3 \\
 u_4 \\
 v_4
 \end{array}$$

We know that

$$\{F\} = [K] \{u\}$$

$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \end{Bmatrix} = [K] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

Applying the boundary Cond

- ① Node 1 & 4 are fixed $u_1, v_1, u_4, v_4 = 0$
- ② Node 2 is roller support $u_2 \neq 0, v_2 = 0$.
- ③ at node 3 a point force N is acting in x dir.
- ④ Body force is zero.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 75 \times 10^3 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 133.2 & -53.28 & 26.64 \\ -53.28 & -133.2 & 0 \\ 26.64 & 0 & 172.05 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_2 \\ 0 \\ u_3 \\ v_3 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 75 \times 10^3 \\ 0 \end{bmatrix} = 1 \times 10^4 \begin{bmatrix} 133.2 & -53.28 & 26.64 \\ -53.28 & -133.2 & 0 \\ 26.64 & 0 & 172.05 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ v_4 \end{bmatrix}$$

Solving the eqn we get

$$u_2 = 0.02766 \text{ mm}, \quad u_3 = 0.067 \text{ mm}$$

$$v_3 = -0.00431 \text{ mm}$$

Stress in each element:

for element 1

$$\sigma_1 = [D][B] \{u\}$$

$$\{\sigma_1\} = 17.78 \begin{Bmatrix} 12 \times 0.067 \\ 3 \times 0.067 \\ 4.5 \times -0.00431 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 14.295 \\ 3.574 \\ -0.345 \end{Bmatrix} \text{ N/mm}^2$$

$$\underline{(\text{element}) \sigma_2} = [D]_2 [B]_2 \{u\}$$

$$\sigma_2 \Rightarrow \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 5.595 \\ 0.2492 \\ 4.196 \end{Bmatrix} \text{ N/mm}^2$$

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} 133.02 & 0 & 24.64 \\ 0 & -23.38 & -133.5 \\ 24.64 & -23.38 & 133.5 \end{bmatrix} \begin{Bmatrix} 133.5 \\ -23.38 \\ 24.64 \end{Bmatrix}$$

global nodal displacements

$$U_2 = 0.000000 \text{ mm}$$

$$U_3 = 0.000000 \text{ mm}$$

Steps in Formulating

$$[D] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 133.5 \\ -23.38 \\ 24.64 \end{Bmatrix}$$